

TWO APPROACHES TOWARD GRAPHICAL DEFINITIONS OF KNOWLEDGE AND WISDOM

Mark Atkins

Florida Institute of Technology, USA

ABSTRACT

Two approaches are taken here in an endeavor to discover natural definitions of knowledge and wisdom that are justifiable with respect to both theory and practice, using graph theory: (1) The metrics approach is to produce graphs that force an increase in various graph metrics, whereas (2) the dimensions approach is based on the observation that the graphical representation of aggelia in the DIKW hierarchy seems to increase in dimension with each step up the hierarchy. The dimensions method produces far more cogent definitions than the metrics method, so that is the set of definitions proposed, especially for use in artificial intelligence.

KEYWORDS

Knowledge Representation, Artificial Intelligence, Graph Theory, DAG, DIKW

1. INTRODUCTION

Despite the emphasis on deriving and using knowledge in our modern era of data mining and applied artificial intelligence (AI), "knowledge" itself has never been well defined. While undefined fundamental terms are fairly common in math and the sciences (e.g., "point," "life," "intelligence"), lack of such definitions often impedes progress. Consider that if one cannot even answer the question "What does it mean for a system to understand something?" then it would be difficult to design a system that understands anything at all, which in fact seems to be the current situation with all known types of computer hardware and software [1] [2], which is clearly impeding progress in applied AI. In addition to immediately practical benefits, such a definition of "knowledge" (and of its more abstract cousin "wisdom") could be of great benefit toward answering much heavier questions, especially in artificial general intelligence (AGI), since such an insight might provide clues about which Knowledge Representation Method (KRM) to use in AGI systems, especially since the KRM of human brains is not known and Edward Feigenbaum once considered this the most important problem in all of AGI [1].

One impediment to establishing such a definition has been the longstanding assumption that knowledge must be true and consistent, even though humans often hold false and inconsistent beliefs despite humans being the paragon of intelligence on this planet. The well-known definition from Plato that "knowledge is justified true beliefs" has recently been shown to be faulty, based on the hypothetical Gettier cases[3]. An underlying reason for the difficulty of determining a useful definition is that knowledge is an intangible, abstract, theoretical construct [3]. A number of interesting metaphors to knowledge have been made [4], especially objects [5],

energy, waves [4], and fluid flows, but these have lacked detailed descriptions or formulas. In general, modern research seems to have concentrated on truth values and metaphors rather than the data structures that might hold those truth values.

This lack of key definitions is somewhat surprising in computer subfields since it is generally understood that directed, acyclic graphs (DAGs) have great representational power, to the extent that many concepts from AI (e.g., expert systems, semantic networks, neural networks, bayesian networks) use DAGs to represent them, as do many concepts from traditional computer science (e.g., flow charts, various automata graphs, Entity Relationship Diagrams, fork-join diagrams, Petri nets, computer network diagrams, precedence diagrams, DeMarco data flow diagrams, logic circuit diagrams), as do the recurring key problems themselves that are common across many scientific fields (e.g., the travelling salesman problem, the subgraph isomorphism problem, the Hamiltonian path problem, the vertex cover problem, the clique problem, the graph coloring problem) that exemplify the P versus NP-problem, which is one of the most important unsolved problems in computer science. This situation is a major clue that if precise, useful definitions of knowledge and wisdom do exist, then those definitions are likely representable by some variation of DAGs. Another clue is that drawing pictures is a well-known problemsolving heuristic in mathematics [6]. These were the two main guiding heuristics that steered this study toward graph theory for a definition.

2. MOTIVATION

This study began in 2008 to provide a practical method to measure the amount of knowledge in knowledge-based systems versus databases for comparison purposes in the commercial software world. Such a method had not existed because practical definitions of the concepts of knowledge and wisdom have not existed (e.g., [7] [8] [9]), a preliminary problem that required solution before any such metric could be developed. The company in Westminster, California that motivated this study folded the same year, whereupon this potentially valuable study became shelved for over a decade.

3. BACKGROUND DEFINITIONS

DIKW hierarchy. The DIKW hierarchy describes the hierarchical relationship of the four main levels of data abstraction: Data, Information, Knowledge, and Wisdom, where Data is at the bottom level of this hierarchy, and Wisdom at the top. Usually this hierarchy is represented as a pyramid called the DIKW pyramid (Figure 1). Although the bottom two levels (Data and Information) of the pyramid are well understood, the top two levels (Knowledge and Wisdom) are not. Since this hierarchy sheds some light on the relationships of the elements in question, it is used as the starting point for this study.

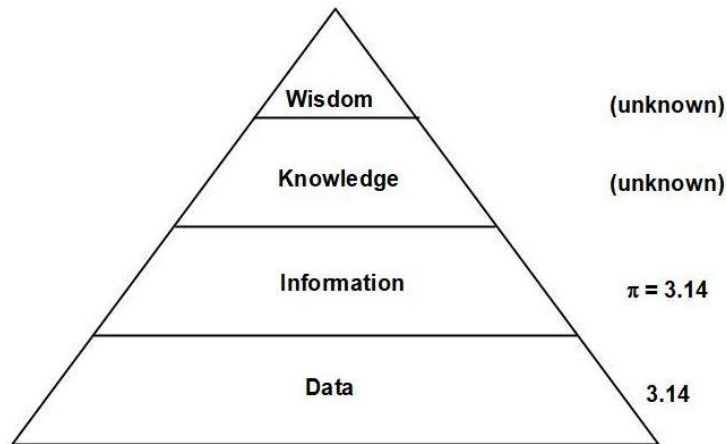


Figure 1. The DIKW pyramid

Aggelia. Let us define "aggelia" (pronounced like "ag-el-EE-ah") as the general type of content in the varied levels of the DIKW hierarchy. In other words, aggelia denotes the information-like content the DIKW hierarchy holds, in a generic sense. Aggelia is an ancient Greek word that means "message" or "announcement."

Dumbbells. Let us define "dumbbell" generically as two vertices of a graph connected by an edge (Figure 2). The edge may or may not be directed, depending on need, and may or may not be labeled or weighted. A single dumbbell with a non-directed edge is equivalent to the simple path P_2 (e.g., [10]), K_2 (e.g., [11]), or S_2 (e.g., [10]), all well-known from graph theory. Dumbbells are often used as building blocks in networks of any kind, especially in networks of knowledge.

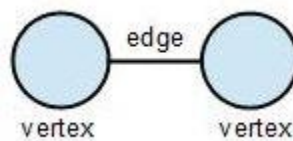


Figure 2. A single dumbbell with an undirected edge

WDAGs. Despite the general lack of understanding of aggelia, software developers in applied AI have pushed forward with temporary conventions about what knowledge and wisdom might look like to a digital computer in order to get practical results immediately. From both rule-based expert systems (RBESs) and neural networks standard data structures have arisen that contain a head, tail, and a numerical weight on the directed edge (Figure 3). In MYCIN this weight is interpreted as the amount of evidence for the assertion described via text at the tail (consequent) [12], and in neural networks this weight is interpreted as the strength of signal transmission to effect the desired mapping from head to tail. In graph theory such a network is called a weighted directed graph. Since applied AI data structures almost always remove or control cycles in networks (e.g., recurrent neural networks, the backpropagation algorithm) we can assume that all such graphs considered here contain no cycles, which then qualifies them as weighted, directed

acyclic graphs (DAGs), most commonly called "edgeweighted DAGs," but sometimes "wDAGs," or "WDAGs". For brevity we use the term WDAG here.

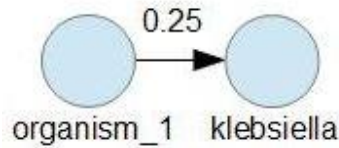


Figure 3. A WDAG is composed of dumbbells such as this.

LDAGs. In semantic nets a similar data structure exists, with the same textual labeling on head and tail that is common in RBESs and neural networks, but with textual labeling instead of numerical values on the edges (Figure 4), which in this case denotes the relationship between the two connected vertices. Let us abbreviate "labeled DAG" as "LDAG."

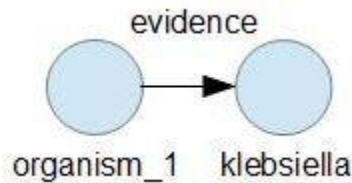


Figure 4. An LDAG is composed of dumbbells such as this.

LWDAGs. If a WDAG is combined with an LDAG then the result could be called an "LWDAG," which is the term used here. Since such a graph is remarkably general, LWDAGs will be the starting point for the modifications discussed here. Modern RBES languages such as CLIPS and JESS use the equivalent of such LWDAGs, although often enhanced by optional calculations in the head (= antecedent). Per graph theory the double edge containing both text and a number prevents the graph from being classified as a simple graph (e.g., [13]) but this can be overlooked in various ways, such as considering the number to be part of the text.

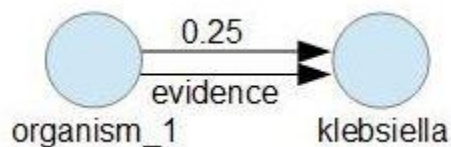


Figure 5. An LWDAG is composed of dumbbells such as this

4. DEVELOPMENT OF THE GRAPHICAL REPRESENTATION METHOD

4.1. Overall Strategy

In general we seek some attribute that would cause a graphical representation of one level of abgaglia to qualitatively "transcend" that layer naturally so that the result would be considered the next higher level of abstraction. This will be called "qualitative transcendence" here. To gain

sufficient insights into the problem to initiate a promising research path we will use the following overall strategy of investigation:

1. make observations from data, information, and (to some extent) knowledge represented as LWDAGs, for initial insights and direction
2. take consideration of the two possible approaches to defining knowledge and wisdom
 - (a) to cause an increase in the values of metrics
 - (b) to increase dimensions
3. if any of those approaches are promising, propose the implied formal definitions

The reasoning for the non-consideration here of other known graph extensions is as follows. **K colored graphs** (e.g., [14]) are those with labels that are consecutive integers, which is a special case of textual labels, which were already selected above in labeled graphs, so the colored graph extension is redundant. **K-partite graphs** (e.g., [14]) have a constrained structure and in Petri nets can be used with added edges for special transition functions that "fire," but a KRM graph would likely have its descriptive power harmed by additional structural constraints, and its transitions have no obvious need to fire. **Marked graphs** are special purpose graphs such as Petri nets with tokens for modeling processes and states, which are a different purpose than representing aggelia as done here, so marked graphs do not extend the power of a graph as a KRM in a useful enough way here. Graphs with inputs, such as graphs representing discrete finite state automata (DFAs) or Turing machines are not needed since the graph is already representing possible input; **graphs with inputs** are mainly used to represent Turing machines, which are special purpose machines to represent states rather than aggelia itself. **Graphs with outputs**, such as Turing machines represented as graphs, specify some type of desired behavior such as printing, but behavior is merely an effect on the outside world that is already being sensed continually, and if there does exist an important adjunct behavior it should be explicitly described by itself.

4.2. Observations from Data and Information Represented as LWDAGs

Data. The nature of data is already well-known. Data typically consist of numbers or text that is unorganized or has no context. For example, the value 3 is data that could be a measurement of some kind, identification of some kind, computer memory address, an order within a list, or something else. Since data itself is typically of more interest than their organization or context, the latter of which are already understood by the implied organization or positions within a data structure, the convention here will be to represent data by vertices of a graph, and to represent relationships by the edges of that graph. In graph theory a single vertex is called an "isolated vertex" (e.g., [14]) or "trivial graph" (e.g., [11]), so an isolated vertex will represent a single datum here. There exists only one obvious method of measuring isolated vertices, and that is to count them, so both diagrams and measurement of pure data are trivial, other than possibly making a decision as to whether duplicated data should affect the count. (See Figure 6.)



Figure 6. amount of data = count of unique isolated vertices = 2

Information. The nature of information is also well-known. As soon as a datum like 3 is given a context such as "x = 3", or designated location of potential importance like "memory[515] = 3", the resulting expression is considered information and no longer data. The obvious resulting representation of information would be a labeled edge connecting the two related data vertices, all of which together become the LDAG defined earlier. (See Figure 7.) Note that even if two or more dumbbells share the same vertex, the counting method for information is the same: only dumbbells are counted, never individual vertices or edges.

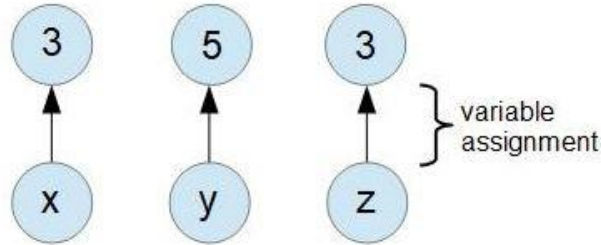


Figure 7. amount of information = count of dumbbells = 3

Knowledge. Clearly, knowledge would tend to be represented by collections of dumbbells. In an RBES these dumbbells may or may not all be connected while the RBES' corresponding rules wait for combinations of states that will fire them (detected by their head), though in a DFA where there exists a small set of all critically important states represented by vertices, all vertices would be connected head-to-tail in a network.

Nature has taught us that epiphenomena commonly arise when a high enough quantity has been reached of the underlying components, such as a high enough quantity of oxygen molecules giving rise to sound, or a high enough film frame rate giving rise to the perception of a moving picture, or a high enough quantity of fissile material giving rise to critical mass that produces a sustained nuclear chain reaction. Thus it is entirely possible that, per popular terminology, a vertex with a high enough count of adjacent vertices might be considered to have "wisdom" rather than merely "knowledge." (See Figures 8 and 9.) More generally, any graph component—such as vertex, edge, dumbbell, group, or border—can have any measurable attribute—such as count, degree, length, distance, or nesting depth—and any applicable summary function can be used—count, average, maximum, minimum, and various ratios of those values. This will be Approach #1.

On the other hand, note that as soon as a few dumbbells have been connected to each other to form a tree structure instead of a list structure, the dimension of the resulting structure usually increases from 1-D to 2-D (except if the resulting graph happens to be a linear chain of nodes like P_n). This suggests that it may not be necessarily the specific number of dumbbells that have been connected that made the essential difference, but rather than they have been connected in such a way that they can no longer fit into their former dimension, since increasing a dimension could certainly be considered a form of qualitative transcendence. This will be Approach #2.

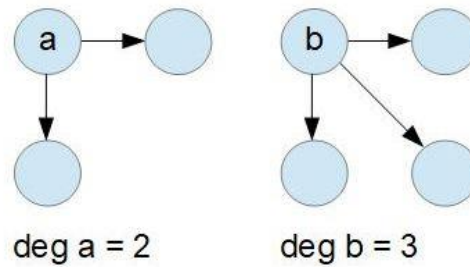


Figure 8. Does the graph on the right contain more "knowledge" than the graph on the left? Any "wisdom"?

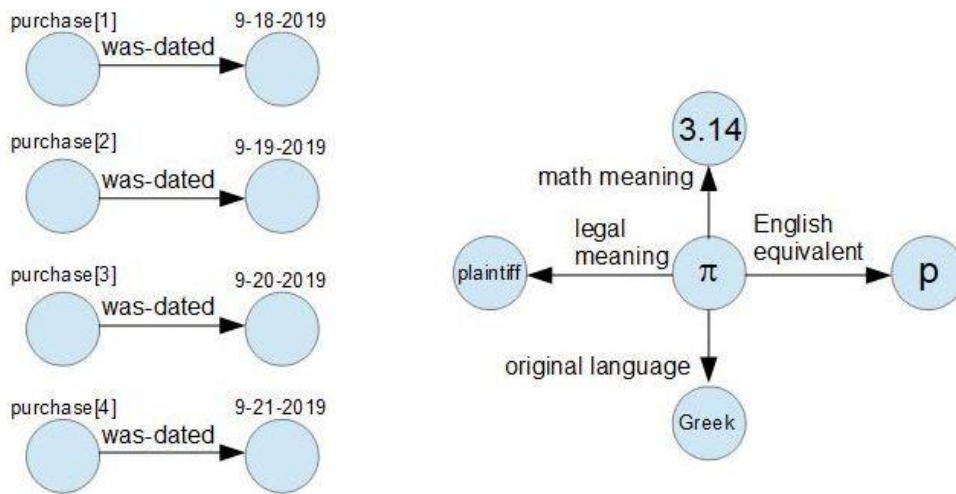


Figure 9. Does the graph on the right contain more "knowledge" than the graph on the left? Any "wisdom"?

5. CONSIDERATION OF THE TWO POSSIBLE APPROACHES

5.1. Approach #1: Increase in the Values of Metrics

If this approach were consistently applied, the following situations would ensue:

1. Enough data, represented as isolated vertices, could be considered information.
2. Enough information, represented as dumbbells, could be considered knowledge.
3. Enough knowledge, represented as a larger graph of dumbbells, could be considered wisdom.

Unfortunately, Situations #1 and #3 outright fail: (Situation #1) Since by definition we know data vertices must be isolated in order to lack associations, and since by definition information must be connected in order to contain associations, then more data can still be only data, not information; one form of aggelia cannot transform to the other via a change in quantity, in this case. (Situation #3) From common usage of the term "wisdom" it becomes clear that likelihoods are normally

involved, even if those values are 100%, and even if the values are not stated. Some examples of folk wisdom are:

- Everyone must pay taxes. (Except the wealthy, or residents of The Bahamas or Bermuda.)
- Money can't buy happiness. (Unless your income is at the low end of the scale, especially in a poorer country [15].)
- (in chess) A knight on the rim is grim. (Except in Petrov's Defense Italian Variation or the Caro-Kann Defense Exchange Variation.)

A piece of wisdom is most commonly a generalization in a very complicated field such as life, business, love, psychology, biology, mathematics, natural language, music, art, or chess, where some heuristic is desirable to make headway among the myriad of possibilities, but where that heuristic is very rarely guaranteed true 100% of the time (else it would likely be a scientific law). This observation strongly suggests that the notion of wisdom equates to heuristics, each of which requires at least a likelihood value, and usually requires a large group of possibilities from which to obtain sufficiently supportive statistics. Since knowledge graphs as discussed so far contain neither groups of dumbbells nor weights on such groups, the common meaning of wisdom cannot apply to knowledge graphs without additional modification, therefore Approach #1 fails on creating wisdom from knowledge.

Although the above knowledge-is-much-information definition (Situation #2) could be forced to work, the fact that two of three such qualitative transcendence patterns do not hold means these definitions are not natural and consistent. Collectively the evidence says that Approach #1 is flawed for usage in natural definitions of transcendence to the next level of *aggelia*. However, the metrics used in Approach #1 will still always be useful for measuring the amount of *aggelia* in any single level of the DIKW hierarchy, if a definition of that given level can be supplied.

5.2. Approach #2: Increase Dimensions

Unlike Approach #1, it turns out that Approach #2 has an absolutely consistent pattern of qualitative transcendence across all levels of *aggelia*. The only trick is to find a way to group an entire section of a graph using a type of mechanism that does not already exist in the existing knowledge graphs, then to connect that group to an outside node via an arc. One solution is to group parts of graphs (or all of each graph) with a border, and then treat the border itself as a head to which a new edge can be attached. Such a border could be informally described as a "swollen head" or "swollen vertex" (Figure 10).

A graph "border" is a concept introduced here that is similar to the concepts of hypergraph and colored graph, but is subtly different from each. A hypergraph contains (hyper)edges that connect more than one vertex, but each hyperedge can connect only to a vertex, not to another hyperedge [16]. A colored graph also groups vertices, and it does so by giving each vertex in the group a color, but this color is not represented by a new entity within the graph to which a connection may be made [17], therefore a group cannot function as a type of vertex. Therefore a new mechanism is needed that not only groups vertices, but creates an edge from that group to another vertex. The border concept is therefore used for this purpose in this article. A "border" will be defined as a boundary line drawn around a set of vertices in a connected graph.

As originally suggested for Approach #2, each step up the DIKW pyramid does in fact now imply an increase of one dimension in aggelia representation when the border construct is allowed; all transitions now show qualitative transcendence as originally desired (Figure 11). In retrospect these dimension-based definitions make perfect sense because of their consistent pattern of achieving qualitative transcendence by creating connected groups of each previous data structure:

- information/dumbbells are connected groups of isolated vertices
- knowledge/graphs are connected groups of dumbbells
- wisdom/groups are connected groups of graphs

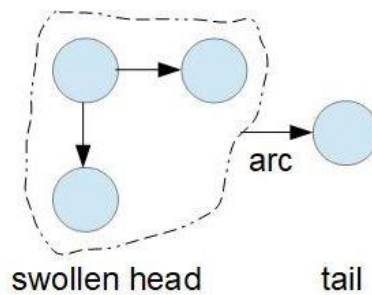


Figure 10. A group could be considered a "swollen head."

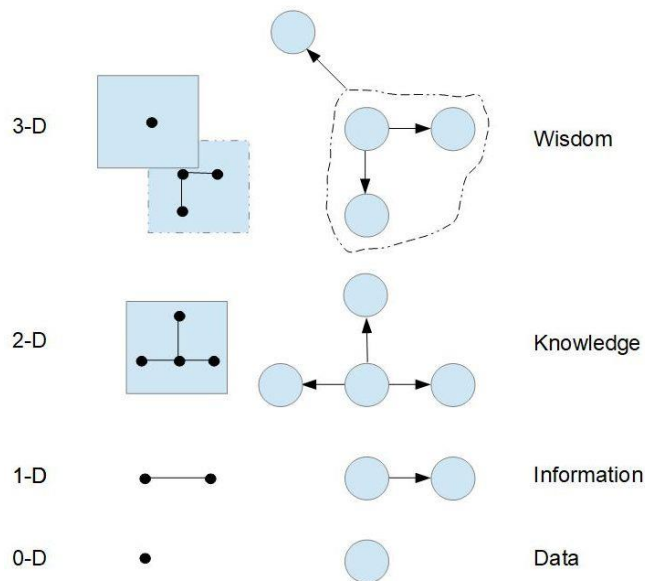


Figure 11. Each step up the DIKW hierarchy suggests an increase in one dimension in aggelia representation.

It appears that over the ages humans have abstractly sensed something qualitatively different about each of the four concepts in the four aggelia levels, viz. the different implied dimensions, to

the extent that humans gave each of those concepts a unique name, even though humans could not define two of the names well. This realization provides appealing psychological support for the proposed definitions. Since graph theory is considered a branch of mathematics (viz., discrete mathematics [18]), the proposed definitions are on solid mathematical ground, which makes the proposed definitions very fortunate for academics, as well.

This completes the two different approaches to defining the four levels of *aggelia*. A trick is used below for the proposed definitions: two sets of terms are used, one set that states mechanically that *aggelia* level x is defined as an "x graphical unit," where x is an element of the set data, information, knowledge, wisdom, then the more second set of definitions states that "x graphical unit" is defined as the graphical unit at the dimension level associated with the proposed graph structure of x . In this way only the second half of the definitions need to be changed if later consensus decided that the proposed graphical associations were unsuitable as a foundation of definitions.

Note that certain fine points are left unresolved in the proposed definitions of this article: (1) whether the pathological path graph P_3 or longer should be considered knowledge because it contains more than two vertices, or whether it should be considered not knowledge because it does need to occupy three dimensions, and (2) whether a non-planar graph should be considered

a 3-D graph only because it cannot be drawn in 2-D without crossing lines (probably it should not be considered 3-D, since its essential character remains unchanged). If the recommended definitions are actually used then practice would best determine the decisions for these questions.

6. IMPLIED FORMAL DEFINITIONS

6.1. General Definitions

Aggelia is the non-physical part of any description. The description can be numerical, textual, pictorial, a data stream, sensor input, a formula, or other. The generality of the description can be at any level. *Aggelia* is the general type of content of the DIKW hierarchy, which includes data, information, knowledge, and wisdom.

A **border** is a single closed loop drawn around selected items within a (usually larger) group to denote that all enclosed items are members of the desired group.

The **Aggelia Definitions Proposal** is a name of the set of proposed definitions in this article, for convenient reference in any later articles

6.2. DIKW-RELATED GRAPHICAL DEFINITIONS

A data graphical unit (DGU) is an isolated vertex.

An **information graphical** unit (IGU) is two units of data that have been associated with an edge, which may be directed, labeled, weighted, or any combination of those modifications. An information graphical unit is also called a dumbbell.

A **knowledge graphical** unit (KGU) is a graph that uses the vertices and edges of dumbbells, where the vertices of any connected dumbbells are shared.

A **wisdom graphical** unit (WGU) is an information graphical unit plus a set of data grouped by border where that group is associated with the data graphical unit via an edge. A wisdom graphical

unit is also called a swollen dumbbell, and the swollen end of it is called a swollen vertex.

6.3.DIKW-Related Aggelia Definitions

Data is aggelia whose structure is that of a data graphical unit.

Information is aggelia whose structure is that of an information graphical unit.

Knowledge is aggelia whose structure is that of a knowledge graphical unit.

Wisdom is aggelia whose structure is that of a wisdom graphical unit.

6.4.Some Useful Proposed Graph Metrics

Assume that G is the graph in question, B is a border in G , and L is a specified label. When measuring the amount of various levels of aggelia in a graph, especially when using ratios and percentages, the following metrics are predicted to work well with the above definitions.

depb(B) = the depth of border B = the count of border crossings for nested borders for B

avgdepb(G) = the average depth of borders in G

bor(G) = the count of borders in G

alab(G) = the count of all unique labels in G

slab(L, G) = the count of a specific label L in G

7. IMPLICATIONS OF THE DEFINITIONS

If the proposed definitions, or some variation of them, became accepted then the theoretical and practical implications would be extensive and important. Some such possible implications are listed below.

Note that one non-implication is influence upon Shannon's information theory, or vice versa. Information theory is based on probabilities and real-life situations (e.g., [19]), whereas the graphical representations here are assumed to be 100% certain. In the current context, to apply probability to the components of a knowledge graph would make as little sense as applying probability theory to a human-designed DFA or to the symbols of a mathematical formula.

7.1. Historical

This would be the first time in over 2,000 years that these concepts, which were debated by Plato and Socrates, were mathematically formalized.

7.2. Terminological

Many frequently used computer terms would be perceived as technically erroneous. For example:

- "databases" would be more accurately called "information bases"
- "data structures" (of computer science) would be more accurately called "information structures"
- "knowledge bases" (of applied AI) with heuristics would be more accurately called "wisdom bases"

7.3. Practical

Objective, numerical comparisons could be made between knowledge base versus knowledge base, or even between more disparate entities such as knowledge base versus database. The proposed metrics, or variations of them, would be used to determine which repository contained more data, information, knowledge, and wisdom, and even the quality of the wisdom.

For example, consider a single table database with 2 records and 3 fields, where the records were instances of people, and the fields were name, phone number, and e-mail address. This database would be equivalent to the following dumbbells with non-directed edges, each dumbbell of which is represented here as an object-attribute-value (OAV) triplet:

(person1 name name1)

(person1 emailaddress emailaddress1)

(person1 phonenumber phonenumber1)

(person2 name name2)

(person2 emailaddress emailaddress2)

(person2 phonenumber phonenumber2)

This database is therefore equivalent to $2 \times 3 = 6$ dumbbells, or 6 IGUs. In general, disregarding compound keys and non-visible indices, a single table database of R records and F fields contains $R \times F$ information graphical units.

As a practical example of how databases can be compared with knowledge bases using the methods of this article, consider Figure 9. In answer to the question posted in the caption of

Figure 9 the unconnected graph on the left that represents a database contains 4 information graphical units, but contains no graphs that require 2-D space, therefore it contains zero knowledge, only information. This is exactly what one might expect from a database. In contrast, although the graph on the right also contains 4 information graphical units, the graph as a whole represents

knowledge—a higher level of abstraction—which is exactly what one might expect from a knowledge base. Neither Figure 8 nor Figure 9 contain any wisdom since neither has any groups.

7.4. An Architecture that "Understands"

The meaning of "understanding" would possibly become completely understood and therefore implementable on a machine. For example, regarding the question in the caption of Figure 8, the answer would be that graph "b" (the graph with vertex "b") does in fact contain more knowledge than graph "a" because graph "b" contains one more dumbbell than the other: 3 instead of 2. To understand why this claim makes sense in a practical case, reconsider Figure 9, which would represent "understanding" of the symbol π . Someone who understood that π could also mean "plaintiff" and not just the well-known mathematical constant would be considered to have better "understanding" of the symbol π —what it can mean and what it can do—and this better understanding would be solely due to the presence of that one extra dumbbell. If all four components of *aggelia* are represented simultaneously as a vector [D I K W], then graph "a" contains [3 2 0 0] graphical units, and graph "b" contains [4 3 1 0] graphical units—one more IGU than graph "a".

A very striking observation is that this example likely generalizes to an extreme degree, to the extent that it may even explain the key to commonsense reasoning. This seems possible because the concept of "understanding" concept x seems to mean only that all immediately relevant attributes of x are "felt" or considered simultaneously as x is considered, and since attributes would be represented graphically as the distal vertices of dumbbells with central vertex x , then if vertex x were active, as in a firing neuron, all the simultaneous associations exactly one arc distant would also be activated (Figure 12). Since neurons have an extreme number of connections, about 104, and since the number of associations necessary to truly understand concept x would also be large, this suggests that a graphical star (e.g., [10]) whose head and tails represented concepts could well implement a neural network capable of true understanding. Since it is clear the brain has an attention focusing mechanism, called an "attentional spotlight" [20], the activated central vertex x of a star would implement this focusing mechanism in a simple way. Similarly, "attentional shift" or "train of thought" would be implemented simply by activation shifting from the vertex x to an adjacent vertex. In other words, understanding happens automatically in a graph with a single node that serves as the attentional spotlight, and the more adjacent nodes the better it understands. Thinking would then likely equate to some type of controlled, goal-directed activation of such concept nodes. These ideas about the meaning of understanding date back to at least year 2000 [21].

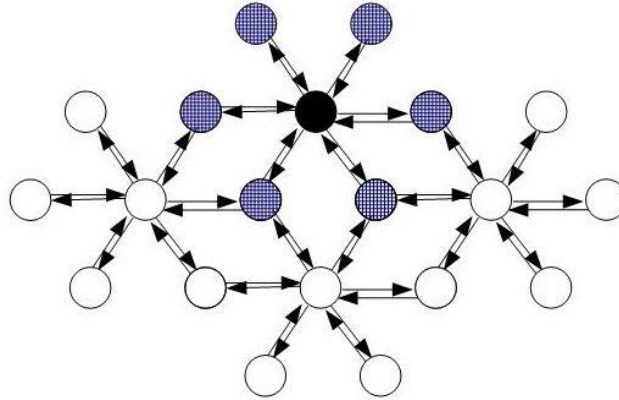


Figure 12. Conjecture: The central node of the active star "understands" that node's semantic concept via simultaneous consideration of all associations that are exactly one arc distant.

7.5. Commonsense Reasoning

As mentioned above, AGI systems would be more likely to be designed that would naturally support commonsense reasoning. This could be extremely important because commonsense reasoning was believed by John McCarthy to be one of the key problems of AGI [22].

It seems clear that some sort of subconscious, neural statistics gathering happens in the human brain. During the process of experiencing the real world, huge volumes of real-world information pass through the brain such that common spatiotemporal patterns are gradually learned, though only subconsciously. The result is a statistical prediction of what will follow in time when an early part of a given spatiotemporal pattern is presented to the brain. For example, if a typical object being held in the air is released under typical conditions, the stored statistics predict that the object will fall. Whatever representation the brain uses for the geometrical translation operation is evidently applied to the current object, and a fast but crude simulation then automatically ensues that predicts the outcome, visually.

The group construct that allows graphical implementation of wisdom, as shown earlier in Figure 10, would be nearly ideal for collecting such statistics. The group itself could include only vertices (neurons) that experienced a given event (e.g., releasing), so the weight on the outgoing arc could be a statistical summary of what percentage of the time the event represented by the tail vertex (e.g., falling) ensued. In effect, the graph "understands" not only concept x that it is currently perceiving, but also "understands" what is likely to happen next. Rather than having a programmer estimate the weight on that arc, the system could estimate that weight based on its own experience (i.e., by counting its own nodes inside that group and calculating the ratio of have-fallen versus have-not-fallen nodes), and better yet might well be able to recall every single event that contributed to that weight, something that a programmer would not likely have the time or ability to encode. In this way wisdom is closely associated with commonsense reasoning, which in turn is closely associated with understanding, and since wisdom would now be understood and therefore implementable, so would understanding and commonsense reasoning, which could lead to a milestone advance in AGI.

7.6. Beyond Wisdom

A natural question to ask when studying the DIKW pyramid is, "Could there exist a level of higher abstraction than wisdom?" Using the graphical methods of this article, the answer is no, because there does not exist any obvious extension that cause wisdom to undergo qualitative transcendence, no matter how components are connected or grouped.

However, certain real-world applications suggest such a higher level might be useful in practice. This situation would arise naturally, for example, if a country's economic flow were modeled with a geographical representation such that instead of points of origin there were regions of origin on a map of that country, which would force the arcs to be 3-D and to change in cross-sectional density to represent the weight at each point in continuous 2-D space. Such 3-D arrows may or may not terminate in a single point representing a concept. However, if such arrows both started and ended in 2-D space, that would create a 4-D mapping, which would require yet another dimension of representation. Whether such an application would justify a new name for a fifth aggelia level is debatable, but since the cerebellum typically stores such seemingly continuous mappings of vectors (e.g., [23]) during creation of its own brand of fine-tuned motor-coordination "knowledge," eventually we would want intelligent machines to be able to do the same thing.

8. CONCLUSIONS

The metrics approach to producing qualitative transcendence in graphical representations of the different levels of aggelia is not consistent, but the dimensions approach of Figure 11 is completely consistent, therefore the increasing dimensions approach's implied definitions of knowledge and wisdom are the definitions proposed here. Those definitions are not only based on solid mathematics in the form of graph theory and dimensions, but are psychologically justified since there exists a qualitative difference between aggelia levels with those definitions.

9. FUTURE WORK

The given example of how to measure aggelia in a 1-table database could likely be extended easily to an n-table database. A complication that might be interesting to pursue for future research is mappings of vectors to vectors, which would create a fifth level of aggelia, a level that would require 4-D representations.

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