COMPARATIVE STUDY ON BENDING LOSS BETWEEN DIFFERENT S-SHAPED WAVEGUIDE BENDS USING MATRIX METHOD

Koushik Bhattacharya[1] and Nilanjan Mukhopadhyay[2]

[1,2]Department of Electronics and Communication Engineering, GIMT, Krishnagar, India
{k.bhattacharyya10, nilu.opt}@gmail.com

ABSTRACT

Bending loss in the waveguide as well as the leakage losses and absorption losses along with a comparative study among different types of S-shaped bend structures has been computed with the help of a simple matrix method.This method needs simple 2×2 matrix multiplication. The effective-index profile of the bended waveguide is then transformed to an equivalent straight waveguide with the help of a suitable mapping technique and is partitioned into large number of thin sections of different refractive indices. The transfer matrix of the two adjacent layers will be a 2×2 matrix relating the field components in adjacent layers. The total transfer matrix is obtained through multiplication of all these transfer matrices. The excitation efficiency of the wave in the guiding layer shows a Lorentzian profile. The power attenuation coefficient of the bent waveguide is the full-width-half-maximum (FWHM) of this peak .Now the transition losses and pure bending losses can be computed from these FWHM datas.The computation technique is quite fast and it is applicable for any waveguide having different parameters and wavelength of light for both polarizations(TE and TM).

KEYWORDS

Excitation Efficiency, S-shape Bend, Transition Loss, Leakage Loss, FWHM, Attenuation Coefficient.

1. INTRODUCTION

Optical integrated circuits (OICs) has gained importance both in terms of performance and cost effectiveness. The size of the OICs is reducing by bending the waveguide channels. The optical loss introduced by these bends is a key factor which determines the overall performance and the achievable density of components of an OIC fabricated on a single substrate[1].There are various bend configurations such as two-corner bends, S-shaped bends, coherently coupled multiple section bends etc. S-shaped bends[1] are widely used in integrated optical circuits because they provide low-loss transitions between parallel waveguides with a lateral offset and are relatively easy to design and fabricate[1].BPM is the most powerful technique to analyze devices with structural variations along the propagation direction, and it provides detailed information about the optical field[1]. However, even 2D-BPM is a highly computation-intensive program requiring huge computer run-time and memory. 3D-BPM applicable to channel waveguides is even more computation-intensive [1]. An approach for analyzing the propagation of electromagnetic waves through a stratified medium is the matrix method [2]. Ghatak et al. [3] presented an analysis of 2D optical waveguides which is computationally fast involving only straightforward

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multiplication of 2x2 transfer matrices of layers. Since this method uses rarely any iteration, it is extremely fast in computation using a standard PC [1].

2. BENDING LOSS AND TRANSITION LOSS

There are different types of losses occurred in the channel waveguide due to bending, they are mainly bending and transition losses shown in Fig. 1[1].The major two types of losses are: [a] pure bending loss due to any curvature in waveguide and [b] transition loss arising from the discontinuity in waveguidecurvature.Total bending loss is the addition of these two losses [1].

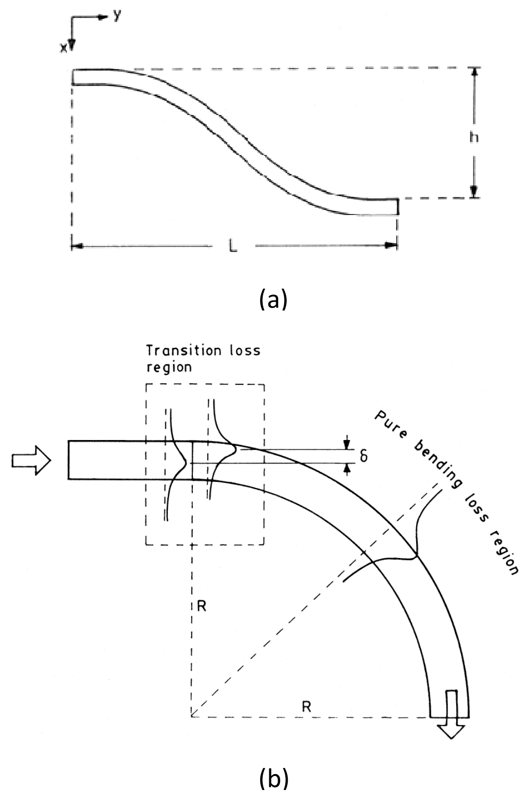


Fig 1. (a)S-type bending structure with transition length *L* and lateral offset *h*[1] (b) The transition and pure bending loss region[1]

The scope of this work is to investigate the light propagation within a channel waveguide and to calculate these losses for mainly three types of S-shaped bends(Double arc, cosine generated and sine generated) and to find an optimized solution.

3. MATHEMATICAL BACKGROUND: MATRIX METHOD

A stratified dielectric medium is considered as shown in Fig. (2).The electric field in each medium for a plane wave incident at angle $\theta_1$may be written in the form [4]

$$\varepsilon_i = \overline{e}_i^+ + E_i^+ e^{i\Delta_i} \exp[i(\omega t - k_i \cos\theta_i x - \beta z)] + \overline{e}_i^- E_i^- e^{-i\Delta_i} \exp[i(\omega t + k_i \cos\theta_i x - \beta z)] \ldots\ldots (1)$$

Where

$$\Delta_1 = \Delta_2 = 0;\ \Delta_3 = k_3 d_2 \cos\theta_3;\ \Delta_4 = k_4 \cos\theta_4 (d_2 + d_4);\ k_i = k_0 n_i = \frac{\omega}{c} n_i$$

and $\beta = k_1 \sin\theta_1 = k_2 \sin\theta_2 = ... = k_4 \sin\theta_4$

βIs an invariant of the system; $\bar{e}_i^+$ and $\bar{e}_i^-$ represent the unit vectors along the direction of the electric field, and E_i^+ and E_i^- represent the electric field amplitudes of waves propagating in the downward and upward directions in i^{th} medium (Fig. 2), respectively [4]. On applying the appropriate boundary conditions at the interfaces obtain

$$\begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix} = S_1 \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} = \cdots = S_1 S_2 S_3 \begin{pmatrix} E_4^+ \\ E_4^- \end{pmatrix} \qquad (2)$$

Where,

$$S_i = 1/t_i \begin{pmatrix} e^{i\delta_i} & r_i e^{i\delta_i} \\ r_i e^{i\delta_i} & e^{i\delta_i} \end{pmatrix} \qquad (3)$$

r_i And t_i represent, respectively, the amplitude reflection and transmission coefficients at the i^{th} interface; $\delta_i = k_i d_i cos\theta_i$ [4]. For TE polarization

$$r_i = \frac{n_i \cos\theta_i - n_{i+1} \cos\theta_{i+1}}{n_i \cos\theta_i + n_{i+1} \cos\theta_{i+1}}$$

and $$t_i = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_{i+1} \cos\theta_{i+1}} \qquad (4)$$

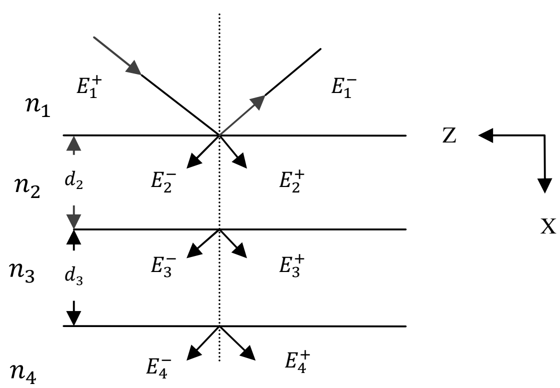


Fig 2. The incidence of a plane wave at an angle θ_1[4]

and for TM polarization

$$r_i = \frac{n_i \cos\theta_{i+1} - n_{i+1} \cos\theta_{i+1}}{n_i \cos\theta_{i+1} + n_{i+1} \cos\theta_{i+1}}$$

and $$t_i = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_{i+1} + n_{i+1} \cos\theta_{i+1}} \qquad (5)$$

Obviously, for a wave incident from the first medium, there would be no upward propagating wave in the last medium, and thus for the structure shown, in (Fig.2), $E_4^- = 0$. Using this condition, and using (2) and (3) the fields are calculated the fields in terms of E_1^-. In order to use the above matrix method in determining the propagation characteristics of planar waveguides, the second, third, and fourth media (in Fig. 2) may correspond to the superstrate, the wave guiding film, and the substrate, respectively [4]. If one now evaluates the excitation efficiency of the wave in the film (i.e. $\eta(\beta) = \left|\frac{E_3^-}{E_1^+}\right|^2$ or $\left|\frac{E_3^+}{E_1^+}\right|^2$) is evaluated as a function of β , then one would obtain resonance peaks appeared [5] which are Lorentzianin shape; the value of βat which the peaks appear gives the real part of the propagation constant and the full widthat half maximum (FWHM) represents the power attenuation coefficient which is just twice the imaginary part of the propagation constant. If 2G represents the FWHM of the resonance peak, then pure bending loss for length *L*'of a constant curvature bent waveguide is given by [1]

B=4.34(2G) L' (dB)………………………… (6)

Once the propagation constant has been determined, the fields throughout the system can be calculated by using the appropriate matrices at the interfaces. This method involves only straightforward multiplicationof 2 X 2 matrices, and no iteration needs to be carried out to determine the real and imaginary parts of the propagation constant, and those accuracies better than one part in a million can readily be obtained [4].
Representing the waveguide bend of (Fig. 1) by an equivalent straight waveguide of length equal to the arc length of the central line of radius of curvature *R*, the effective refractive index of the equivalent waveguide along the transverse axis x may be expressed as [6],

$$n_{eq}(x) = n_{eff}(x)(1+\frac{x}{R}) \quad \text{.......................... (7)}$$

$n_{eq}(x)$ Is the transverse effective index profile of the original waveguide without any curvature.The fundamental mode of the waveguide can be well approximated by a Gaussian distribution of the form [1]

$$E(x) = E_0 \exp(-\frac{x^2}{2a_x^2}) \quad \text{........................... (8)}$$

The modal offset between two waveguides of radii of curvature R_1 andR_2 is then given by [7],

$$\delta = (\frac{V_1^2 a_x^4}{\rho^2 \Delta})(\frac{1}{R_1^2} - \frac{1}{R_2^2}) \quad \text{........................... (9)}$$

It is evident from Eq. (9) that the modal offset increases with increasing difference in curvature. In Eq. (9), ρ is the waveguide half-width and

$$\Delta = \frac{n_{\max}^2 - n_{sub}^2}{2n_{\max}^2} \quad \text{,.................................... (10)}$$

$$V_1 = \frac{2\pi}{\lambda} \rho n_{\max} \sqrt{2\Delta} \quad \text{.. (11)}$$

Where$n_{max}$is the maximum refractive index value on the surface of the waveguide andn_{sub} is the refractive index of the substrate at the wavelength, λ[1]. The spot size$a_x$appearing in Eq.(8). And (9) may be calculated from an Eigen value equation of the form [8]

$$\text{Exp}\,({}^{1}/_{a_x^2}) = 2A_x \frac{V_1^2}{\sqrt{\pi}}, A_x = (\frac{a_x}{\rho}) \text{.................... (12)}$$

Since in almost all practical cases, this modal offset is a small quantity, the transition loss can be expressed as [6]

$$T = -4.34\ln\left(1-\frac{\pi^2\delta^2}{4W^2}\right) \text{ dB}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(13)$$

Where W is the waveguide width.

Next, the model is used to calculate the bending losses of more complicated S-shaped channel waveguide bends (Double arc, cosine generated and sine generated).The corresponding curvature variations along the three different S-curves are [1],

$$\frac{1}{R_1(y)} = \frac{2\pi h}{L^2}\sin\left(\frac{2\pi y}{L}\right) \text{(For sine generated)} \ldots\ldots\ldots\ldots\ldots\ldots.. (14)$$

$$\frac{1}{R_2(y)} = \frac{\pi^2 h}{2L^2}\cos\left(\frac{\pi y}{L}\right) \text{(For cosine generated)} \ldots\ldots\ldots\ldots\ldots.. (15)$$

$$R_3(y) = \pm\frac{L^2}{4h}\left(1+\frac{h^2}{L^2}\right) \text{(For double arc)} \ldots\ldots\ldots\ldots\ldots\ldots...... (16)$$

Where h is the lateral offset between the two parallel sections and L is the transition length in the longitudinal direction (fig.1).

5. Computational Result

When light incident on the first medium at the guiding medium is excited by the portion of that light wave. So to study the excitation efficiency as a function of incident angle is the key thing for every computation. The nature of the plot is same for TE and TM mode.

5.1. Computation of Excitation Efficiency for TE mode

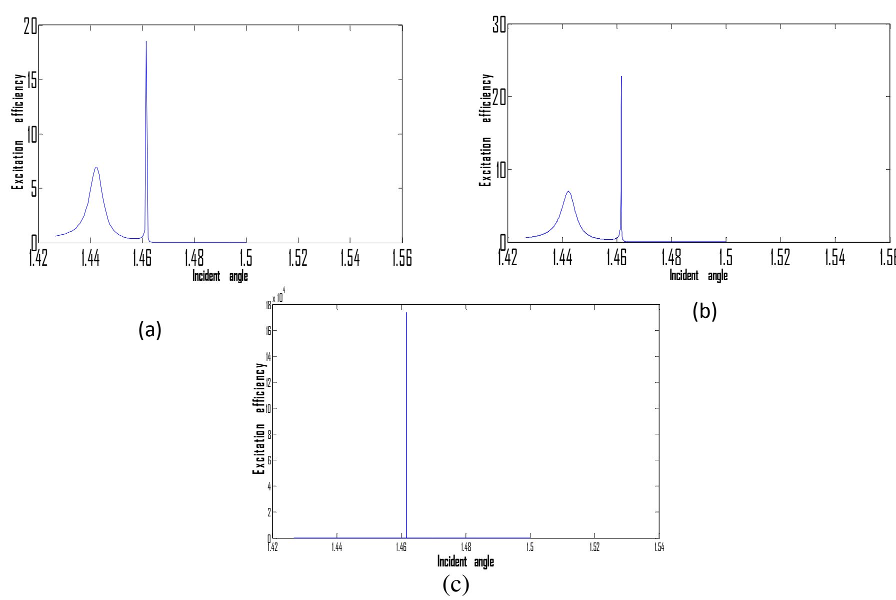


Fig: 4. Variation of excitation efficiency with angle of incidence at first medium for TE mode for resolution of incident angle (a) 10^{-4}(b) 10^{-5} and (c) 10^{-6} with refractive indices for first medium $n_1 = 1.5$, second medium $n_2 = 1.4$, third medium $n_3 = 1.5$, forth medium $n_4 = 1.4$ and the thickness of the layers as $d_1 = 10\mu m$, $d_2 = 0.2\mu m$, $d_3 = 2\mu m$.

For TE mode the resonance occurring at the guiding medium at a particular incident angle. At this incident angle the excitation efficiency becomes large. Resolution of incident angle is taken to be10^{-5}. All the results are obtained with the help of MATLAB (version 7.5) software.

Similarly for TM mode the same results for the resonance occurring at the guiding medium for a particular value of the incident angle can be obtained.

5.2. Effect of increasing the thickness of superstrate (d_2)

The thickness of the superstrate i.e. $d_2$has an effect on the excitation efficiency. If the thickness is increased from 0.3 to 0.6 then the amplitude of the excitation efficiency for the leakage mode goes on decreasing and the guided mode becomes more confined as shown below.

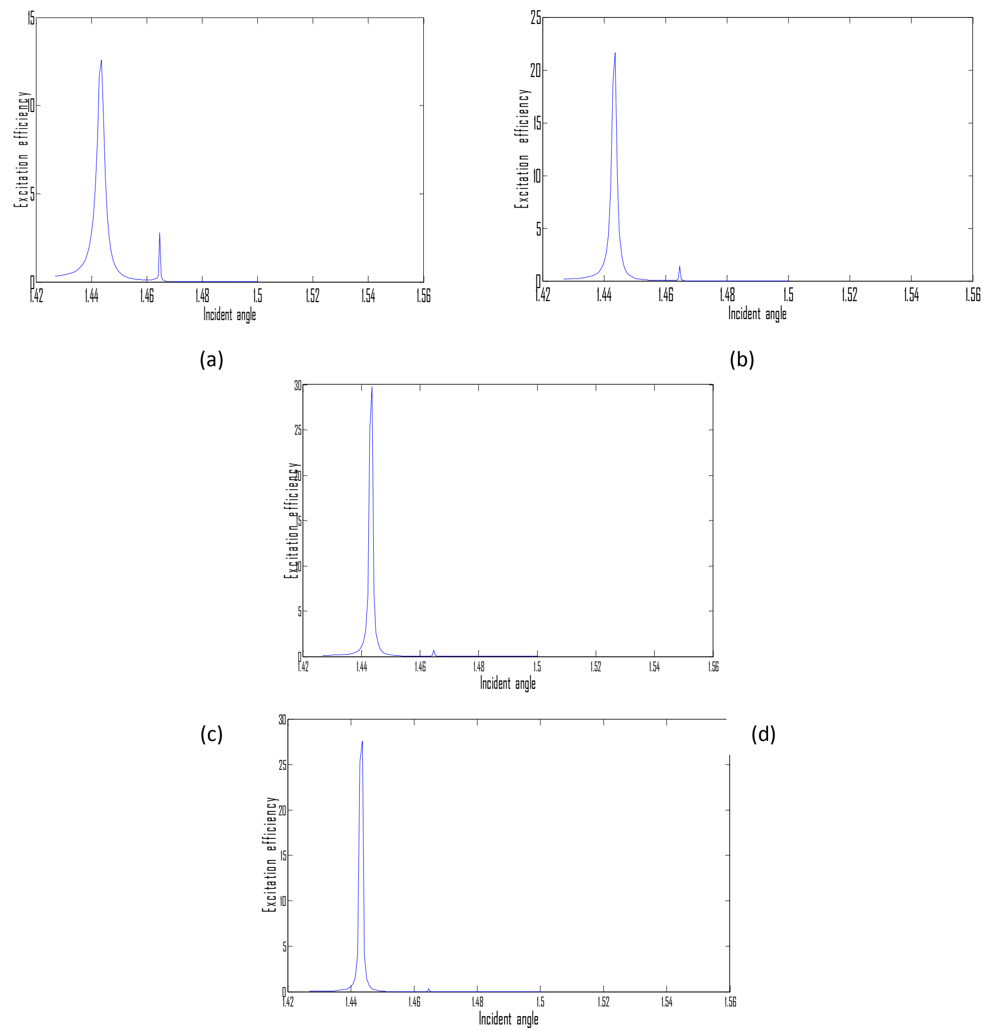


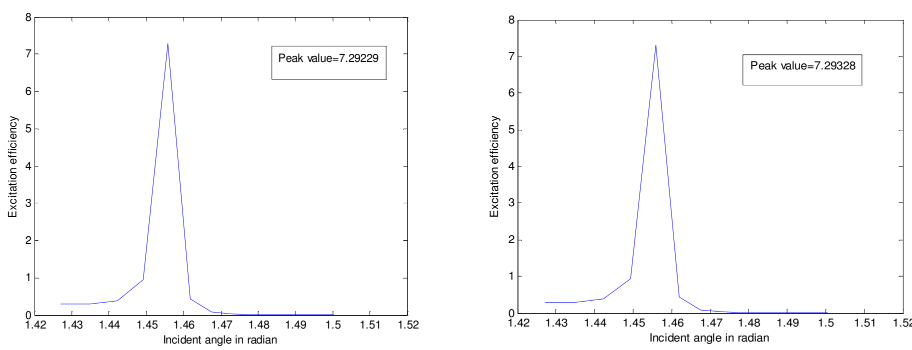
Fig: 5 Variation of excitation efficiency with thickness of superstrate for (a) $d_2 = 0.3\mu m$(b) $d_2 = 0.4\mu m$ (c) $d_2 = 0.5\mu m$and (d)$d_2 = 0.6\mu m$

5.3. Effect of bending the waveguide with a bending radius(R)

In a bended waveguide if the thickness of the superstrate is increased then the amplitude of the excitation efficiency reduced and the excitation efficiency as a function of incident angle yields a Lorentzian shaped resonance curve. The value of βat which the peak appears gives the real part of the propagation constant of the guided waves, and the full width at half maximum FWHM represents the power attenuation coefficient which is twice the imaginary part of the propagation constant.

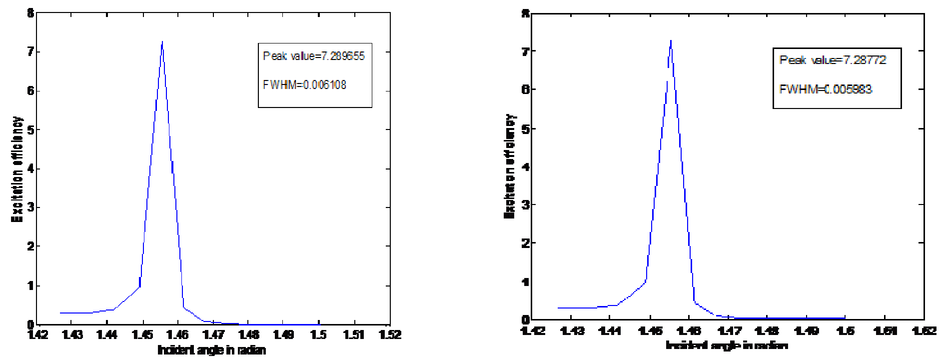
5.3.1. Calculation of pure bending loss fora double arc and cosine-generated *S*-shaped bend waveguide

The pure bending loss for double arc and cosine-generated S-shaped bend structure has been computed here. For the double arc is a structure having two radius of curvature with one value positive and other is negative. Here the value of the radius of curvature has been kept same. The radius of curvature for the double arc and cosine-generated structure follows Eqn.16 and Eqn.15 respectively. The bending loss can be calculated using Eqn.6.Depending on the variation of bending radius (R) and the waveguide length (L') the pure bending loss also changes.



(a) (b)

Fig: 6 Excitation efficiency as a function of incident angle for the computation of pure bending loss as (a) R=7.2mm, L'=1.2mm, calculated FWHM=0.006980 radian (for positive arc) and (b) R=8.5mm, L'=1.3mm, calculated FWHM=0.006916 radian (for positive arc).



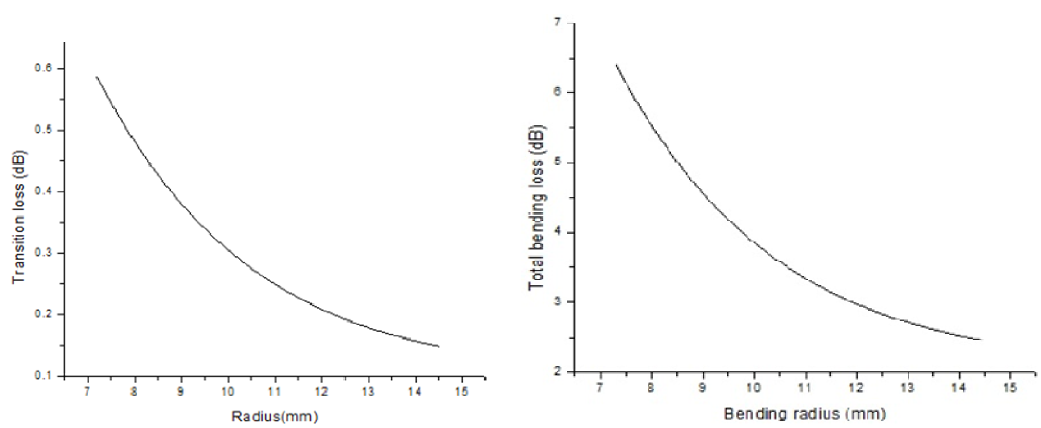
(a) (b)

Fig: 7 Variation of pure bending loss for cosine-generated bend waveguide as a function of transition length and bending radius for different transition length (L) as (a) L=500µm (b) L=1000µm.Values of the parameters are taken as h (lateral offset) =100µm and λ=1µm.Calculated FWHM are given with the figure itself.

From the above results it is cleared that the amplitude of the excitation efficiency decreases as the bending radius increases.The FWHM is calculated from each profile and then with the help of Eqn.6 the pure bending loss can be calculated. It may be noted that there is a rapid increase of bending loss if the radius of curvature decreases below 10 mm.

5.3.2. Transition loss and total bending loss as a function of bending radius for an S-curve made of two circular arcs

Variation of the radius of curvature has the effect on transition loss as well as total bending loss as observed from the following results. The transition loss is calculated with the help of Eq. (13).



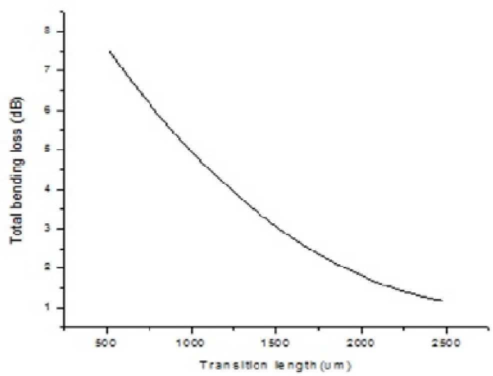
(a) (b)

Fig: 8 Variation of (a) transition loss and (b) total bending loss as a function of bending radius with the required parameters values W (waveguide width)=2µm,ρ (waveguide half width)=1µm and λ=1µm.

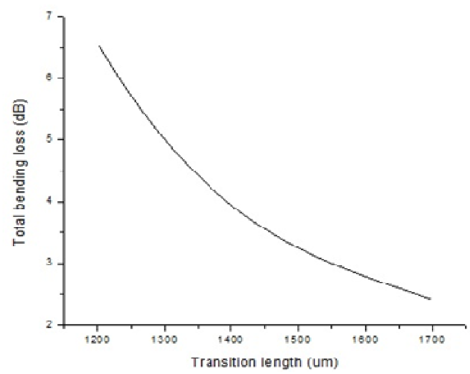
Total bending loss is calculated by adding the transition loss and the pure bending loss. From the above two plot it can be seen that the transition loss is less than the pure bending loss. Also the transition loss increases slightly below R=10mmcompared to total bending loss. For sine and cosine generated structures the same variation can be obtained.

5.3.3. Comparison of total bending loss between different *S*-shaped bends waveguide as a function of transition length

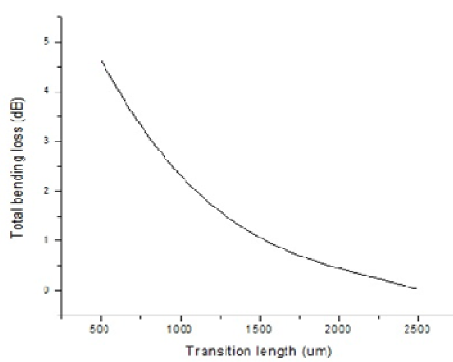
Now with the help of the transition loss calculated from the Eqn.13, the total bending loss can be obtained by adding that with the calculated pure bending loss. This total bending loss changes with the variation of transition length of the waveguide as found from the below results.



(a)



(b)



(c)

Fig:9 Variation of total bending loss as a function of transition length for (a) double arc (b) sine- generated (c) cosine-generated S-shaped bend waveguide with the required parameter values taken as h(lateral offset)=100µm and λ=1µm.

It can be observed that all the loss profiles are of same nature. The results shows that the total bending loss decreases with the transition length but amount of total bending loss is much less in case of cosine-generated S-shaped bend waveguide than the other two bend structures. So in case of OIC design the use of cosine-generated bend structure is very handful.

6. APPLICATIONS

This method has been vastly used in OIC design. In case of designing a directional coupler the bending losses can be computed and optimized by following the efficient bend structure through this method. With the help of this method it is possible to design couplers for any level of coupling. Other applications involve designing of distributed Bragg reflector (DBR) laser, distributed feedback (DFB) lasers and other passive and active optical devices [12].

7. CONCLUSIONS

To calculate the losses due to waveguide bending, a simple method including matrix multiplication is shown here. Solution of any transcendental or differential equation is not required here. Also a comparative study among three different S-shaped waveguide bend structures (double arc, sine generated, cosine generated) on the basis of total bending loss is shown here and it has been found that the suitable method to minimize the loss is to use the cosine generated S-shaped bend structure. The transition loss has to be taken into account for this computation. For any wavelength and polarization of input light, this technique is applicable. Hence this method is very handful in OIC design. Amplifier can be used to compensate the losses due to bending; but it would be expensive. If the refractive index of the guiding medium can be modified so that it will compensate the losses then it can be a very useful method. This method can be implemented to minimize the losses in channel waveguide by index matching. It does not involve any approximations and thus gives exact numerical results for TE and TM modes. Also the computation using the present model is very fast.

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AUTHORS

Koushik Bhattacharyya is a 4th year student of Electronics and Communication Engineering. He is pursuing his M.Tech from Global Institute of Management & Technology (GIMT), Krishnagar, Nadia.

Nilanjan Mukhopadhyay has received the B. Sc. degree in Physics from Calcutta University, India in 2006. Then the B.Tech & M.Tech degrees were received from Calcutta University in 2009 & 2011 respectively. From 2011 to 2012 he was working in Tata Consultancy Services, Bangalore, India. Since Aug, 2012 he has been working as an Assistant Professor in the Department of Electronics & Communication Engineering, Global Institute of Management & Technology, Krishnagar, India.