INTRODUCING SIMPLEX MASS BALANCING METHOD FOR MULTI-COMMODITY FLOW NETWORK WITH A SEPARATOR

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ABSTRACT

Maximization of flow through the network has remained core issue in the field of optimization. In this paper a new advanced network problem is introduced, i.e., a multi-commodity network with a separator. This network consists of several sub-networks linked through a commodity separator. The objective is to maximize the flow of the commodity of interest from the commodity mixture at the output of the commodity separator, while observing the capacity constraints of all sub-networks. Such networks are present in Oil and Gas development fields. Such networks are also conceptual representation of material flows of many other manufacturing systems. In this paper an algorithm is developed for maximization of flow in these networks. Maximization of flow in such networks has direct practical relevance and industrial application. The developed algorithm brings together two distinct branches of computer science i.e., graph theory and linear programming to solve the problem.

KEYWORDS

Graph Theory, Linear Programming, Multi-commodity Network Flow Optimization, Commodity of Interest, Hybrid Algorithm

1. INTRODUCTION

Flow maximization through networks has been a major problem under study for last several decades [1]. This is because many real world problems can be formulated as a network problem such as optical networks [2], wireless networks [3], reliability networks [4, 5], biological networks [6], production assembly networks [7] and social networks [8] etc.

A typical flow network is a directed graph with number of nodes connected through number of edges. Each edge has a limited capacity. A network also has a source node and a sink node. It is assumed that source node can produce flow of unlimited capacity. The problem is to push maximum flow through the network from the source node to the sink node such that capacity of

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any edge is not violated and all nodes must be balanced nodes i.e. flow going into the node is equal to flow going out of the node.

In 1956 a remarkable theorem on this network was developed which is popularly known as max-flow-min-cut theorem [9, 10]. According to this theorem maximum flow through the network is equal to minimum cut of the network. The minimum cut of the network is defined as a cut of minimum size through the network that disconnects completely the source from the sink such that no flow from the source could pass to the sink. If this cut consists of only single point then any bottleneck in this cut can cause it to become single point of failure (SPOF) triggering failure of the entire system [11]. This theorem might not be applicable in some networks with multiple commodities coming from multiple sources and going into multiple sinks [12], however this theorem provides strong base to construct max flow theorems for many of these types of networks [13]. Based on this theorem a number of approaches have been discovered to solve this problem. These approaches can be divided into two main branches, i.e. augmentation paths algorithms, [9, 10, 14, 15, 16, 17, 18, 19] and pre-flow push algorithms [20, 21, 22, 23, 24, 25, 26, 27, 28]. Some novel ideas have also been discovered such as pseudo flows [29], and draining algorithm [30].

There is also an advanced network problem called multi-commodity flow network problem. A multi-commodity network is the network carrying mixture of commodities from multiple sources. The sources are considered to be of unlimited capacity, each producing a mixture of commodities with different proportions. The problem is to maximize the flow of the commodity of interest (COI) through the network such that final mixture coming out through the sink has the maximum proportion of the COI. This multi-commodity network problem is closely related to oil and gas development field where there is a number of wells connected to a network. Each well produces mixture of oil, gas, and water in different proportions. Industry usually wants to maximize the flow of oil through the network as oil is considered the most precious commodity. A quick solution to this problem was presented in mass balancing theorem [31].

However above problem presents only half of the picture of real world. In the real world industrial scenario, there is a multi-commodity network that terminates onto a separator that separates all these commodities, each of which flows through its own flow network towards its respective sink. Faults occur regularly in these huge networks, which directly affect capacity of the relevant sub-network requiring readjustment of production from source to maximize the output of the COI. Therefore, capacity of an individual commodity network has direct effect on the production system of the whole mixture of commodities. This problem is also conceptual representation of material flows of many other manufacturing systems such as mining Industry. The mining industry has to deal with a raw material coming from different sources. The industry doesn’t have much control over contents of this raw material as each source produces raw material of its own configuration i.e. its own mix of compounds in different proportions. The industry has to process and refine this raw material for further use. However industry has limited processing and refining capacity due to industrial, operational, qualitative and environmental reasons [32]. Therefore the industry needs to determine production rates from different sources in such a way that requirements of its processing unit regarding volume and predefined content of incoming material are met. A similar problem was solved by iterative mass balance method [33] but in that method capacity constraints of network were not considered therefore the method fails short of real world scenario.

In this paper network with a commodity separator problem is formally introduced and its solution is also proposed. The proposed method combines two distinct fields of Computer Science i.e. linear programming [34] and graph theory [35]. Graph algorithms are usually considered faster and simpler than linear programming in the area of flow networks. This is because each node of the network adds one equation to the set of equations to be solved under linear programming. Therefore linear programming becomes very cumbersome with very large networks. On the other hand, graph theory does not have mathematical apparatus to deal with separation of commodities in the network. Therefore, the proposed method makes use of both linear programming and graph theory to achieve easy and quick solution.

The scenario of the network as presented above represents dynamic situation where faults occur and get repaired very frequently. For such a situation a quick method of optimization based on Mass Balancing Theorem was introduced. In mass balancing theorem an interesting property of network was discovered that the fully saturated network is actually balance of certain easily computable flow load on the either side of the minimum cut. Utilizing this property a flow dissipation algorithm was developed to maximize the flow through the network. An interesting thing about this algorithm is that it visits only unbalanced nodes rather than the whole network to maximize the flow. Therefore this algorithm has very important role to play in dynamic networks where the network continues changing its state. A change is marked by removal of E^- edges and/or addition of E^+ edges. Due to these changes each time certain number of nodes becomes unbalanced. The upper bound on this number δ is given by:

$$\delta = 2(E^+ + E^-) \quad (1)$$

The number in equation 1 is only a small fraction of total number of nodes in the network and by visiting only those unbalanced nodes flow can be maximized. Furthermore this algorithm is also extended to the multi-commodity network, where a COI was maximized in presence of multiple sources. However, as in this paper advanced problem of multi-commodity network with a separator is considered. For this advanced problem as discussed earlier the method only based on graph theory is not enough. In such problems flow cannot be maximized without add of linear programming to deal with the separation of flows into individual commodities. Keeping above situation in mind a method has been devised that is hybrid of simplex method based on linear programming and mass balancing method based on graph theory for the particular problem formulated in this paper. However roles of both mass balancing theorem and simplex method are chosen in a way to utilize positive points of both the methods. Flow through all the sub-networks is maximized using mass balancing theorem while simplex method is applied only on a commodity separator to optimize the flow of COI. The hybrid algorithm is called simplex mass balancing (SMB) Method.

The remainder of the paper is organized as follows. Section 2 presents mathematical formulation of the problem, section 3 explains the SMB method, section 4 provides the proof of optimality, in section 5, a solved example of proposed method is presented. Section 6 analyses the complexity of the proposed algorithm, and finally section 7 concludes the paper and discusses the future work.

2. PROBLEM FORMULATION

Let the multi-commodity network consists of n sources and m commodities, and each source j produces a unique mix of commodities in quantity Q^j such that

$$\forall_{j=1,n} Q^j = \sum_{i=1}^{i=m} q_i^j = \sum_{i=1}^{i=m} \gamma_i^j Q^j \qquad (2)$$

q_i^j = flow of commodity *i* in source *j*
γ_i^j = proportion of flow of commodity *i* in source *j* such that

$$0 \le \gamma_i^j \le 1 \qquad (3)$$

Equation 2 shows that total quantity of mixture is sum of all the quantities of individual commodities in the mixture, where quantity of each individual commodity can be determined from its proportion in the mixture. The value of proportion varies between 0 and 1 (expression 3).

Flow from all the sources ultimately terminate onto a separator, where the commodity mixes are separated. At the output of the separator, there are *m* commodity networks each corresponding to a single commodity, carrying i^{th} commodity to the i^{th} sink. The goal is to maximize the output of commodity of interest (COI) while obeying the capacity constraints of multi-commodity network and each of the *m* commodity networks.

Figure 1 shows a multi-commodity network, namely, N_0 connected to *n* sources S_1…… S_n and a separator *U*. In addition, there are *m* commodity networks, namely N_1…..N_m, which originate from the separator *U* and each of these networks has its own sink, i.e. T_1 through T_m, respectively. For the problem formulation, following subsections define some notions.

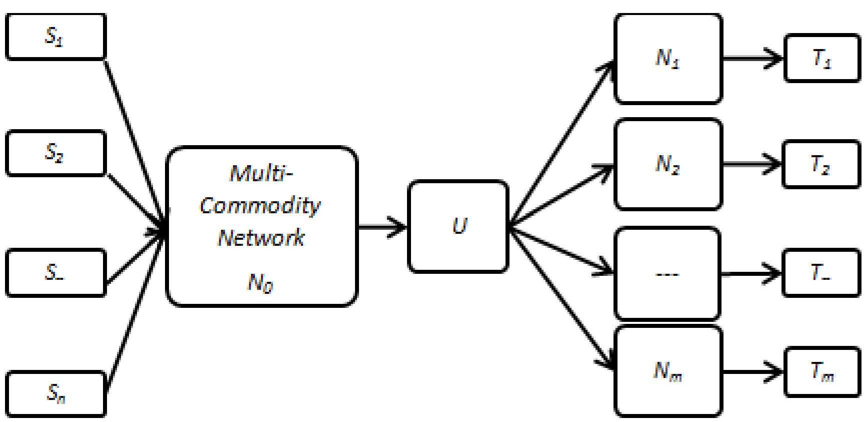

Figure 1. Multi-Commodity Network with Separator

2.1. Unified and Individual Source Networks (USN and ISNs)

Let us modify the network in Figure 1 by connecting its source nodes S_1…… S_n with the universal source node S_0 of unlimited capacity through the edges E_1, E_2, ……, E_n of unlimited capacity respectively. Furthermore considering the separator as the ultimate sink, the network of Figure 1 can be reduced to the network shown in Figure 2. The network hereby referred to as Unified Source Network (USN). The USN in Figure 2 can be calibrated into the individual source networks. The individual source network corresponding to the source i, ISN_i is the network with capacity of $E_i = \infty$ and capacity of $E_j = 0$ where $j \in \{1, \ldots, n / j \neq i\}$. This means that in the individual network of source i all the other sources will be disconnected from the network except source i itself. Furthermore the capacity of source i is also considered unlimited.

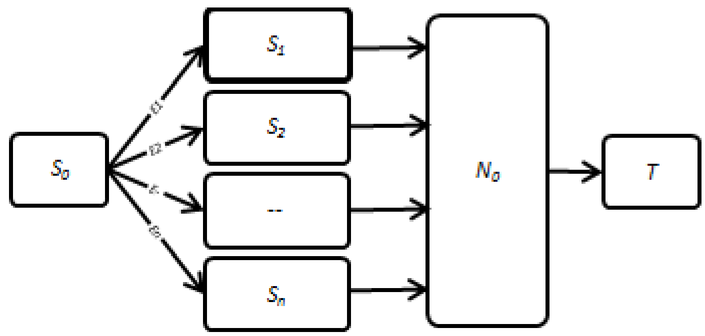

Figure 2. Unified Source Network and n Individual Source Networks

2.2. Individual Commodity Network (ICN)

Considering the separator U as the primary source for each commodity network, the network of Figure 1 can be reduced to the network shown in Figure 3. In Figure 3 since there are m commodities hence there are m ICNs, such that for ICN_i, capacity of $e_i = \infty$ and capacity of $e_j = 0$ where $j \in \{1, \dots\dots, n/j \neq i\}$. This means that in the individual commodity network of commodity i all the other commodities are disconnected from the network except commodity i itself. Furthermore the capacity of primary source of commodity i is also considered unlimited.

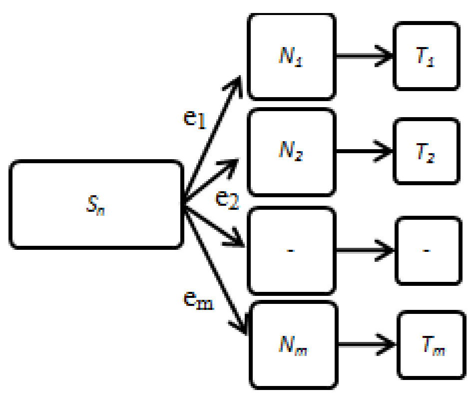

Figure 3. m Individual Commodity Networks

Let us consider

C_0 = Minimum cut of the USN in Figure 2
$C_{si\in\{1,\dots,n\}}$= Minimum cut of the $ISN_{i\in\{1,\dots,n\}}$ respectively in Figure 2
$C_{ci\in\{1,\dots,m\}}$ = Minimum cut of the $ICN_{i\in\{1,\dots,m\}}$ respectively in Figure 3

Therefore the maximum flow Q_0 in the multi-commodity network of Figure 1 is given by

$$Q_0 \leq min\left(C_0, \sum_{i=1}^{i=n} C_{si}, \sum_{i=1}^{i=m} C_{ci}\right) \quad (4)$$

This means that maximum flow in the network can be only be minimum of the following three quantities.

1. Minimum cut of the unified source network
2. Sum of minimum cuts of individual source networks
3. Sum of minimum cuts of individual commodity networks

Since in any case

$$C_0 \leq \sum_{i=1}^{i=n} C_{si} \tag{5}$$

Therefore expression 4 reduces to

$$Q_0 \leq min\left(C_0, \sum_{i=1}^{i=n} C_{ci}\right) \tag{6}$$

Expression 6 shows that maximum flow through the multi-commodity network of Figure 1 cannot be greater than lesser of the two quantities i.e., minimum cut of USN of Figure 2, and sum of minimum cuts of all the individual commodity networks ICNs of Figure 3. The ≤ sign in this expression indicates that there are other constraints too that may restrict the flow. Those constraints are shown in expressions 7 and 8. Suppose q_i^j is flow of commodity *i* in source *j* then

$$\forall_{j=1,n}\, Q^j \leq C_{sj} \tag{7}$$

and

$$\forall_{i=1,m}\, Q_i = \sum_{j=1}^{j=n} q_i^j \leq C_{ci} \tag{8}$$

Expression 8 shows that flow from any source *j* must not be greater than minimum cut of its individual source network and expression 7 shows that total flow of any commodity *i* must not be greater than the minimum cut C_{ci} of its respective individual commodity network. The ≤ sign signifies the fact that if one of the commodities k exhausts the capacity of its individual network C_{ck}, then flow cannot be further increased for other commodities $i \in \{1, \dots, m / i \neq k\}$ as increase in the overall mixture would also increase the flow of the commodity *k*. Therefore objective is to maximize the flow of COI, Q_{coi} i.e.,

$$max(Q_{coi}) = max\left(\sum_{j=1}^{j=n} q_{coi}^j\right) \tag{9}$$

Substituting the values of q_{coi}^j from expression 2 into expression 9 gives the following linear function

$$Z = \sum_{j=1}^{j=n} \gamma_{coi}^j Q^j \tag{10}$$

The linear function in equation 10 is to be maximized under the constraints of expressions 6 through 8.

3. SMB METHOD FOR MULTI-COMMODITY NETWORK WITH A SEPARATOR

A method for maximization of flow of a commodity of interest through the network with a separator has been devised by keeping problem formulation presented in section 2 in mind. The method hybridizes two distinct fields of Computer Science i.e., linear programming and graph theory as explained in section 1. Linear programming is used to maximize linear function shown in equation 10 under the constraints in expressions 6-8. However to determine the value of constraints mass balancing method of flow maximization is chosen. The reasons for choosing this method has already been discussed in section 1. This hybridized method is termed as simplex mass balancing (SMB) method.

The algorithm is explained in the following steps.

1. Create all the networks including USNs, ISNs (Section 2.1) and ICNs (Section 2.2)
2. Maximize the flow through each USNs, ICNs and ISNs to determine the values of x_0, $x_{i\in\{1,....,n\}}$, $y_{i\in\{1,....,m\}}$ respectively, corresponding to n sources and m commodities.
3. Design linear programming formulation (equation 10) under the constraints 6-8 from output of step 2.
4. Maximize the linear function of equation 10 using Simplex method to determine $Q^{j\in\{1,...,n\}}$.
5. Maximize the flow through the USN by equating capacity of $E_{j\in\{1,2,....,n\}}$ with $Q^{j\in\{1,...,n\}}$ respectively.
6. Compute quantity of each commodity q_i using equation 10 from output of step 4.
7. For all i, maximize the flow in ICN_i by equating capacity of $e_{j\in\{1,2,....,m\}}$ with $q_{j\in\{1,...,m\}}$ respectively.
8. Join USN obtained from step 5 and ICNs obtained from step 7 and remove additional edges and universal source node to represent actual network with maximized flow of COI.

In steps 2, 5 and 7 flow is maximized using mass balancing theorem. From the above procedure it can be seen that in in the second step a method based on graph theory is used to determine minimum cuts of various conceptual networks introduced in section 2. The values of those minimum cuts are later used in design of linear programming formulation in step 3. The designed linear function is then optimized in step 4 using simplex method to determine optimal flows from all sources. In step 5, a flow through multi-commodity network is again maximized by restricting flow from sources to optimal flows obtained in previous step. In step 6, quantity of each commodity is computed in the resultant output mixture from all the sources. In step 7, flow in each commodity network is maximized by restricting commodity quantity obtained in previous step. In the final step, all the conceptual networks are joined together to form original network.

4. PROOF OF OPTIMALITY

Simplex method is a well-known method which optimizes linear function [34]. SMB method uses this method to maximize flow of commodity of interest from the mixture of commodities. If the LP formulation is correct then SMB method produces optimal solution. The correctness of LP formulation depends on the correctness of the design of constraints. There are three constraints involved herein.

A. Capacity constraint on the total flow from all the sources
B. Capacity constraint on the flow of individual source
C. Capacity constraint on the flow of individual commodity

According to Ford Fulkerson theorem [9] capacity constraint of any flow network can be computed by determining the size of its minimum cut. On the other hand, size of the minimum cut can easily be determined by applying any well-known flow maximization method on the network. Therefore above 3 constraints can easily be determined by applying mass balancing method on the respective networks. Now if the design of the respective networks is correct then estimation of constraints and corresponding LP formulation would also be correct. Thus final solution obtained through SMB method is the optimal solution. The following are the proofs that the respective networks designed in the SMB method are correct.

4.1. Lemma 1: Minimum cut of USN (Figure 2) = constraint A

Proof: If all the source nodes are turned into junction nodes and a single source of unlimited capacity joins all those junction nodes through connecting edges of unlimited capacity, i.e., capacity of $E_i = \infty$, where E_i = Set of all the connecting edges then maximum flow through this network represents maximal flow through original multiple source network. This was proved in minimum cut theorem [9]. However simpler explanation is produced here and that explanation is also used later in Lemma-2 and Lemma-3 in support of their proof. Consider a single path network in Figure 4. Now suppose that G_i represents capacity of edge E_i such that

$$G_i = min(G_0, G_1, G_2, \dots, G_n) \quad (11)$$

According to equation 11 edge E_i has minimum capacity in all the edges from the source S to sink T. Therefore E_i represents minimum cut C_{min} of this network and the maximum flow that could pass through this network cannot be greater than the capacity of edge E_i, i.e.

$$C_{min} = G_i \quad (12)$$

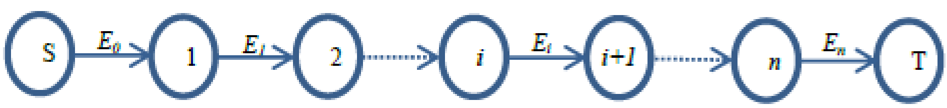

Figure 4. A single path network

Now suppose if the source node in Figure 4 is turned into a junction node and then this node is connected with another source of unlimited capacity through edge E_j as shown in Figure 5.

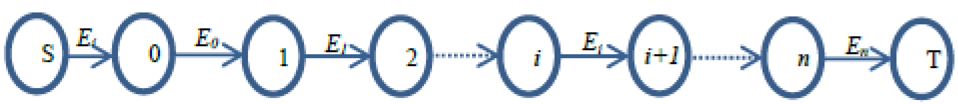

Figure 5. An extended single path network

Now it is very obvious from new network that

$$\begin{cases} C_{min} = G_j & if \quad G_j < G_i \\ C_{min} = G_i & otherwise \end{cases} \quad (13)$$

Now if edge E_j is considered of unlimited capacity i.e., $G_j = \infty$ then according to condition (13), equation 12 remains true even after the modification in the network.

Since in USN the same modification is incorporated therefore minimum cuts of all paths from sources to sink remain unaffected, and hence minimum cut of overall network remains the same, thus it represents a bottleneck capacity for flow from all the sources in a multi-commodity network.

4.2. Lemma 2: Minimum cut of ISN (Figure 2) = constraint B

Proof: Suppose USN (Figure 2) consists of n connecting edges of unlimited capacity i.e. capacity of $E_i = \infty$ and $E_i \in \{E_1, E_2, E_3, \dots, E_n\}$ then as proved in Lemma-1 any flow maximization algorithm on this network determines maximum capacity constraint on flow from all the sources S_1, S_2, S_3, ….., S_n or determines minimum cut of the network connecting all these sources with the separator/sink.

Now according to definition of ISN as explained in section 2.1, for the ISN_i of source S_i capacity of $E_i = \infty$ and capacity of $E_j = 0$ where $j \in \{1, \ldots\ldots, n/j \neq i\}$, then according to condition 13 minimum cut of all ISN_j emanating from set of sources S_j becomes zero. However minimum cut of ISN_i emanating from source S_i remains unaffected. Hence the ISN_i can be used to determine maximum flow from S_i to sink/separator.

4.3. Lemma 3: Minimum cut of ICN = constraint C

Proof: Here set of edges of unlimited capacity i.e. capacity of $e_i = \infty$ is created, connecting the separator/source with each commodity network. Then by definition in section 2.2, ICN_i for commodity i can be created such that capacity of $e_i = \infty$ and capacity of $e_j = 0$ where $j \in \{1, \ldots\ldots, n/j \neq i\}$.

Now according to condition 13, minimum cut of all ICN_j emanating for set of commodities j becomes zero. However minimum cut of ICN_i emanating for commodity i remains unaffected. Hence the ICN_i can be used to determine maximum flow from source/separator to sink T_i.

5. SOLVED EXAMPLE

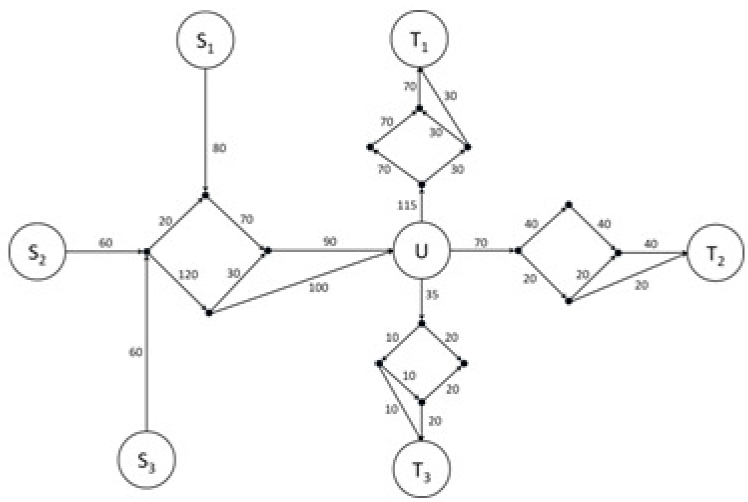

Figure 6. The example of multi-commodity network with a separator

Table 1. Source Configuration

	S_1	S_2	S_3
C_1	0.6	0.5	0.4
C_2	0.3	0.2	0.5
C_3	0.1	0.3	0.1

Figure 6 consists of network with separator. It has three sources S_1, S_2, & S_3. Each source has 3 commodities. Ratio of each commodity in each source is given in Table 1. Consider commodity C_1 as a commodity of interest which needs to be maximized.

The network in Figure 7 represents USN if $E_1 = E_2 = E_3 = \infty$. Since there are 3 sources hence there must be 3 ISNs in Figure 7. By definition of ISN, in section 2.1., in Figure 7, ISN_1

constitutes $E_1 = \infty$ and $E_2 = E_3 = 0$, ISN_2 constitutes $E_2 = \infty$ and $E_1 = E_3 = 0$ and ISN_3 constitutes $E_3 = \infty$ and $E_1 = E_2 = 0$.

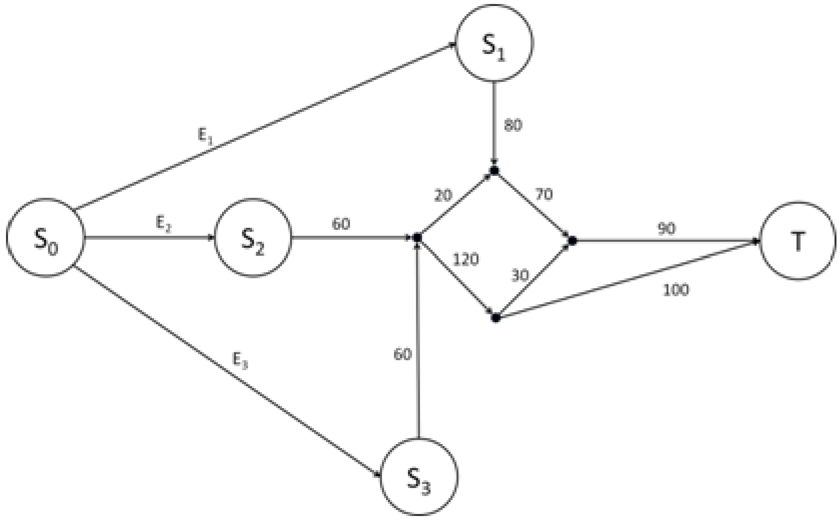

Figure 7. USN and ISNs of the example of Figure 4

Since there are 3 commodities in the network there must be 3 ICNs as shown in Figure 8. By definition of ICN in section 2.2, in Figure 8 ICN_1 constitutes $e_1 = \infty$ and $e_2 = e_3 = 0$, ICN_2 constitutes $e_2 = \infty$ and $e_1 = e_3 = 0$ and ICN_3 constitutes $e_3 = \infty$ and $e_1 = e_2 = 0$.

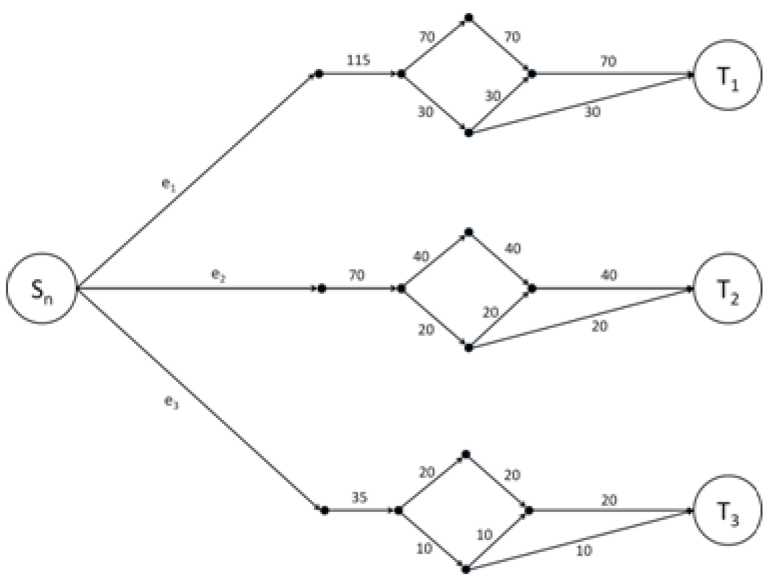

Figure 8. ICNs of the example of Figure 4

With formulation of Figure 7 and Figure 8 step 1 of algorithm has been completed i.e. USN, ISNs and ICNs have been created. In step 2 of algorithm flow is to be maximized in all these networks to determine their minimum cuts to develop LP formulation. Let C_0 be the minimum cut of the USN. After applying flow maximization algorithm on the USN of Figure 7 we get C_0=190. Let

C_{si}, be the minimum cut of ISN$_i$. After applying flow maximization algorithm on ISNs of Figure 7 we get C_{s1}=70, C_{s2}=60 and C_{s3}=60. Let C_{ci} be the minimum cut of ICN$_i$. After applying flow maximization algorithm on ICNs of Figure 8 we get C_{c1}=100, C_{c2}=60 and C_{c3}=30. In step 3 of the algorithm LP formulation is developed from this data as follows.

Maximize the linear function

$$Z = 0.6x1 + 0.5x2 + 0.4x3 \tag{14}$$

Under the constraints

$$\begin{Bmatrix} 0 \le x_1 \le 70 \\ 0 \le x_2 \le 60 \\ 0 \le x_3 \le 60 \end{Bmatrix} \tag{15}$$

$$\begin{Bmatrix} x_1 + x_2 + x_3 \le 190 \\ 0.6x_1 + 0.5x_2 + 0.4x_3 \le 100 \\ 0.3x_1 + 0.2x_2 + 0.5x_3 \le 60 \\ 0.1x_1 + 0.3x_2 + 0.1x_3 \le 30 \end{Bmatrix} \tag{16}$$

Where x_i = flow from source *i*

In Step 4 of the algorithm above linear function is maximized through simplex method and following solution is obtained

$$\begin{Bmatrix} x_1 = 70 \\ x_2 = 58.4615 \\ x_3 = 54.6154 \end{Bmatrix} \tag{17}$$

In step 5 of the algorithm flow is maximized through the USN by equating E_1, E_2 and E_3 of Figure 7 with x_1, x_2 and x_3 respectively.

In step 6 of algorithm, output of each commodity is computed from the results of equation 17 as follows.

$$\begin{Bmatrix} y_1 = 0.6 \times 70 + 0.5 \times 58.4615 + 0.4 \times 54.6154 = 93.07691 \\ y_2 = 0.3 \times 70 + 0.2 \times 58.4615 + 0.5 \times 54.6154 = 60 \\ y_3 = 0.1 \times 70 + 0.3 \times 58.4615 + 0.1 \times 54.6154 = 30 \end{Bmatrix} \tag{18}$$

Where y_i = flow of commodity *i*

In step 7 of the algorithm we maximize flow through the each ICN by equating e_1, e_2 and e_3 of Figure 8 with y_1, y_2 and y_3 of equation 18 respectively.

In the final step the USN obtained in step 5 is joined with ICNs obtained in step 7 and all the added edges E_1, E_2 and E_3 along with the universal source node is removed to represent actual network with maximized flow of COI. The final solution is shown in Figure 9.

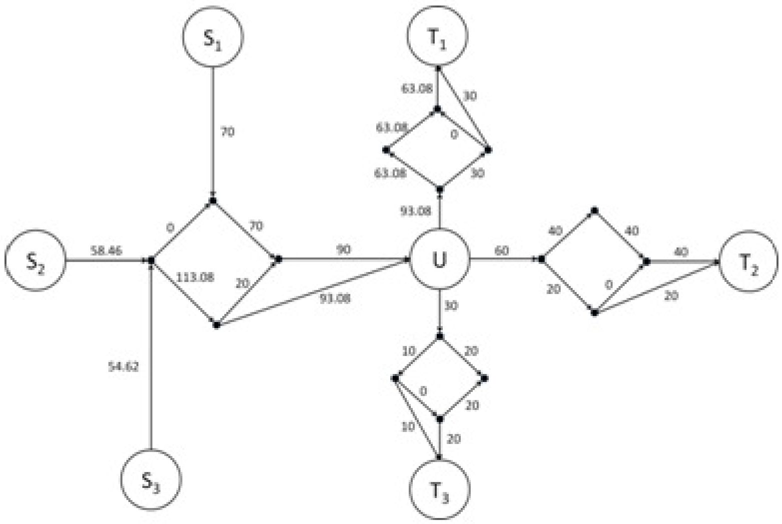

Figure 9. Final solution of Network with Separator

6. COMPLEXITY ANALYSIS

The complexity ω of mass balancing theorem (MBT) has already been established [31] and i.e., of order $O(m^2 - m)$ where m is the number of edges. It can be seen that MBT procedure is applied twice on USN (Step 2, 5) twice on ICN (Step 2, 7), and once on ISN (Step 2). Therefore total complexity of MBT procedure on network with separator is given by:

$$\omega = (2 + s)(m_1^2 - m_1) + 2t(m_2^2 - m_2) \quad (19)$$

where

m_1 = number of edges in USN and ISN
m_2 = average number of edges in ICN
t = the number of commodities
s = number of sources

In case of very large networks having tens of thousands of edges, the number of sources and number of commodities would become meaningless thus complexity reduces to:

$$\omega = (m_1^2 - m_1) + (m_2^2 - m_2) \quad (20)$$

Since this complexity can never be greater than $O(m^2 - m)$, where m is the total number of edges in the multi-commodity network with a separator i.e. $m = m_1 + m_2$. Therefore, complexity of MBT method for network with separator stands the same as that of standard network with one source and one sink. However to compute overall complexity of SMB method, complexity of simplex method should also be added into this. However simplex method depends only on number of commodities rather than the network size, hence again it will become meaningless in large networks. Therefore complexity of SMB method stands at $O(m^2 - m)$ only.

7. CONCLUSION AND FUTURE WORK

This paper introduces a problem of flow maximization through a multi-commodity network with a separator and describes its applications in oil and gas development fields and mining industry among others. The paper also presents a method to maximize flow through this network, hereby referred to as the Simplex Mass Balancing (SMB) method. The proposed method uses combination of two most important but distinct branches of computer science i.e. linear programming and graph theory. The computational cost of the SMB method is small because linear programming is applied only on the flow input from the sources and flow output from the separator not including the network, while mass balancing theorem is applied on the networks only for flow maximization to determine the constraints needed for LP formulation. This combination has resulted in optimal solution with very less computational cost. The proof of optimality has also been formulated. The future direction of proposed work can be its extension to more complex problems like network with multiple separators.

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